

Inductive reasoning, domain specific and complex problem solving: Relations and development



Gyöngyvér Molnár^{a,*}, Samuel Greiff^b, Benő Csapó^a

^a Institute of Education, University of Szeged, 30-34. Petöfi S sgt., Szeged, H-6723, Hungary

^b Department of Psychology, Heidelberg University, Hauptstraße 47-51, Heidelberg, D-69117, Germany

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ABSTRACT

This paper focuses on three different types of reasoning: domain-specific problem solving, complex (general) problem solving, and inductive reasoning. The objective of the study is to examine the differences in the developmental levels of inductive reasoning, domain-specific problem solving, and complex problem solving between three age groups and to describe the relations between the three constructs. The sample was drawn from 3rd to 11th grade students (aged 9–17) in Hungarian primary and secondary schools. There were 300–400 students in each cohort. The internal consistencies of the tests were good: Chronbach α varied between .72 and .95. Each of the skills showed a developmental tendency that could be identified with a logistic curve. In every area the pace of development proved to be relatively slow and the steepest change took place in Grade 7. The bivariate correlations between the three constructs were moderate ranging from .35 to .44 signalling that they do not constitute the same construct. The strength of the relationships between inductive reasoning and complex problem solving proved to be the most stable over time. The correlations between domain-specific and complex problem solving showed an increasing trend over time indicating that the strategies used in different problem solving situations become more similar with age. This study provides evidence that inductive reasoning, domain-specific problem solving and complex problem solving are related but distinct constructs and these skills can be fostered most efficiently between Grades 6 and 8.

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1. Development of thinking skills

Fostering the development of thinking skills has long been considered one of the most important educational goals (Resnick, 1987). In theory, two main groups of thinking skills are distinguished: Skills closely related to specific domains (e.g. domain specific problem solving; Schoenfeld & Herrmann, 1982) and general thinking skills applicable in a variety of different contexts (e.g. complex problem solving, see Frensch & Funke, 1995; inductive reasoning, see Klauer & Phye, 2008). In practice, there is no sharp distinction between the two sets of skills, because measurement of a specific thinking skill is always embedded in some kind of content, and requires the application of general mental processes as well (Ericsson & Hastie, 1994), even though the twofold conceptual distinction is important when understanding the cognitive processes and the development underlying these thinking skills. Beyond doubt, thinking skills are tools to success in today's society characterized by rapid change, where the nature of applicable knowledge changes frequently and specific contents quickly become outdated (de Konig, 2000).

* Corresponding author. Tel.: +36 20 77 56 478; fax: +36 62 420 034.
E-mail address: gymolnar@edpsy.u-szeged.hu (G. Molnár).

Measures of thinking skills are rooted in intelligence research, which has dedicated a lot of attention to the role of inheritance: Early views considered thinking skills fixed and immutable (Jensen, 1973). Recent studies, on the contrary, indicate that thinking skills develop over time and that their development spans several decades; moreover, they are modifiable offering opportunities for enhancement by specifically designed and targeted educational interventions (Adey, Csapó, Demetriou, Hautamaki, & Shayer, 2007). Thus, the description of their development is relevant not only for theoretical reasons, but from a practical perspective as well, first of all, because the stimulation of thinking skills is most efficient when their development is still in progress, especially in the fast-growing phases (Csapó, 1997).

Problem solving skills (PS) have been extensively studied over the last decade as they are seen as the most broadly applicable cognitive tools. Developing problem solving skills is a major objective of educational programmes in several countries (OECD, 2010). To this end, a consistent research finding is that problem solving depends on domain-specific knowledge and strategies (e.g. Mayer, 1992; Funke & Frensch, 2007); however, problem solving skills also involve the ability to acquire and to use new knowledge, or to use pre-existing knowledge to solve novel problems (i.e., problems that are non-routine; Sternberg, 1994). In the present study, our focus is twofold: First, we will relate three different types of reasoning to each other: domain-specific problem solving, complex (general) problem solving, and inductive reasoning. Second, we will describe their development over time by using cross-sectional data.

2. Inductive reasoning, domain-specific problem solving and complex problem solving

2.1. Inductive reasoning and its development

Inductive reasoning (IR) is a general thinking skill (Pellegrino & Glaser, 1982; Ropo, 1987), which is related to almost all higher-order cognitive skills and processes (Csapó, 1997), such as general intelligence (Klauer & Phye, 2008), problem solving (Gentner, 1989; Klauer, 1996; Tomic, 1995), knowledge acquisition and application (Bisanz, Bisanz, & Korpan, 1994; Hamers, De Koning, & Sijtsma, 2000), and analogical reasoning (Goswami, 1991). Nevertheless, there is no universally accepted definition of IR even though several definitions have been proposed (e.g., Klauer, 1990; Osherson, Smith, Wilkie, Lopez, & Shafir, 1990; Sloman, 1993; Gick & Holyoak, 1983). The exact psychological mechanisms underlying IR, however, are yet to be discovered. A classical interpretation of IR views it as the process of moving from the specific to the general (Sandberg & McCullough, 2010). That is, IR is described as the generalization of single observations and experiences in order to reach general conclusions or derive broad rules (rule induction). We view IR as a general cognitive ability and adopt Klauer's theory (1993) interpreting IR as the discovery of regularities through the detection of similarities, dissimilarities, or a combination of both, with respect to attributes or relations to or between objects (Klauer, 1993).

Empirical studies examining IR from a developmental perspective are scarce at best (Sandberg & McCullough, 2010; Goswami and Brown, 1989). In particular, research on samples of a broad age-range (e.g., Lunzer, 1965; Levinson & Carpenter, 1974) is difficult to find. According to the few available studies, IR develops mainly during elementary and secondary education with the average pace of development being relatively slow at about one quarter of a standard deviation per year (Csapó, 1997; Molnár & Csapó, 2011). This suggests that fostering IR is not an integral part of school curricula (de Konig, 2000), however it can be developed effectively and to a significant extent (Klauer & Phye, 2008; Molnár, 2011). In the absence of direct and explicit stimulation of IR in schools, development occurs spontaneously as a 'by-product' of teaching ordinary school material rather than being guided by explicit instruction (de Konig, 2000).

2.2. Domain-specific problem solving and its development

Different definitions and theoretical models of problem solving have been proposed in the literature (for an overview see Sternberg, 1994), most of them sharing a common aspect, namely, that a problem is characterized by a gap between the current and the goal state with no immediate solution available (Mayer & Wittrock, 1996).

As this gap can be found in any content domain, the research field of domain specific problem solving (DSPS) studies these processes in a variety of settings (Sugrue, 1995): either in connection with specific national school curricula (e.g. Mullis et al., 2009), or explicitly in a specific domain such as mathematical (Daniel & Embretson, 2010), technical (Baumert, Evans, & Geiser, 1999), or scientific (Dunbar & Fugelsang, 2005) problem solving or problem solving in game playing (Frensch & Sternberg, 1991). One of the most comprehensive international large-scale assessments, the Programme for International Student Assessment (PISA), places special emphasis on PS processes and measured it in 2003 as an innovative domain complementing the traditional school subjects of reading, science and mathematics (OECD, 2004). In line with other research findings (e.g. Nickerson, 1994), the level of problem solving skills in PISA is shown to be closely related to domain-specific knowledge and strategies (e.g. Mayer, 1992). Thus, problem solvers need to combine knowledge acquired in and out of the classroom to reach the desired solution by retrieving and applying previously acquired knowledge in a specific domain. In the present study, we treat DSPS as a process of applying domain specific – especially mathematical – knowledge in three different types of new situations: (1) complete problems (all necessary information to solve the problem is given at the outset), (2) incomplete problems relying on missing information that is expected to have been learned at school, (3) incomplete problems relying on missing information that was not learned at school.

Development of PS was a central area of research for the information-processing approach to cognition. This approach focuses on how students progress from being novices to becoming experts within specific domains (see Mayer, 2008) and

how they change their strategies over time (Riley, Greeno, & Heller, 1982). The level of proficiency in solving domain specific problems depends on the problem solvers' domain specific knowledge, including its factual, conceptual, procedural, and strategic aspects (Kilpatrick et al., 2001).

2.3. Complex problem solving and its development

According to Sternberg (1994), some of the aforementioned PS research focusing on the comparison of experts and novices overemphasizes domain specific processes thereby disregarding the impact of domain general processes. A different approach to problem solving, one that uses tasks that can be solved by anybody (i.e., also by students considered to be novices) and focuses on domain general processes instead of content knowledge and rote learning, was taken up in connection with complex problem solving (CPS; see Funke & Frensch, 2007; Funke, 2001; Greiff, Wüstenberg, & Funke, 2012).

According to a widely accepted definition of CPS, problem solvers in CPS situations are confronted with tasks that are novel, unknown, dynamically changing over time, ill-structured, knowledge intensive, and non-transparent (Buchner, 1995; Dörner, 1986). That is, when solving complex problems, the problem solver needs to use domain general processes largely uncontaminated by specific content schemata (Novick, Hurley, & Francis, 1999) because understanding the structure of novel problems is more effective when relying on abstract representation schemas rather than on specifically relevant example problems (see e.g., Holyoak, 1985; Klahr, Triona, & Williams, 2007). According to our interpretation, CPS is a specific form of problem solving in complex and interactive situations, which enables us to study knowledge acquisition and knowledge application simultaneously and independently of specific content. Dealing with such complex and interactive situations requires problem solving skills beyond those involved in DSPS (e.g., Klieme, 2004; Wüstenberg, Greiff, & Funke, 2012; Greiff et al., 2013). That is, CPS additionally requires a series of complex cognitive operations (Funke, 2010; Raven, 2000).

2.4. In what way are inductive reasoning, domain-specific problem solving, and complex problem solving related?

Most of the studies highlighting the relationship between IR, DSPS and CPS do not draw a distinction between DSPS and CPS; their focus is on IR and problem solving in general (see e.g. Sternberg, 1994). According to these studies, IR plays a major role in problem solving (Chi et al., 1982; Johnson-Laird, 1983; Klauer, 1989; Klauer, 1996; Polya, 1954), and in connection with problem solving in hypothesis generation, and hypothesis testing (Gilhooly, 1982). In early information-processing models of problem solving, a central role is attributed to rule induction, which can be considered the essence of IR (Egan & Greeno, 1974; Simon, 1974).

Contemporary cognitive research reveals that IR is applied in information processing during problem solving (Mayer, 1998). It has an effect on the success of knowledge acquisition and application (Bisanz et al., 1994; Pellegrino & Glaser, 1982; Klauer, 1990; Klauer, 1996; Hamers et al., 2000); therefore, IR is essential for gaining a deeper understanding of any subject matter and its application in real-life problem situations. Some studies suggest (see e.g. Klieme, 2004; Wirth & Klieme, 2004) that skills typically used in solving domain-specific problems are different from the knowledge acquisition processes used in exploring a problem in a complex problem solving situation (OECD, 2010). Albeit comprehensive research on the relationship between IR, DSPS, and CPS is scarce, they are expected to correlate as they all involve cognitive abilities indispensable for generating and applying rules and yet they are expected to exhibit unique variance to a certain extent.

3. Aims and research questions

The objective of this study is twofold. First, we examine (1) the differences in developmental levels of IR, DSPS and CPS between three age groups. We then examine (2) the relationships between IR, DSPS and CPS. More specifically, this study endeavours to outline the developmental trends of IR, DSPS, and CPS estimating the age range when major development takes place, and locating sensitive periods, in which explicit training and/or modified school instruction are expected to have the strongest effect. Further, the study undertakes to describe the relationships between IR, DSPS and CPS in general and as they change over time.

We thus intend to answer three research questions: (1) How do IR, DSPS and CPS develop over time during compulsory schooling? (2) What is the role of IR in the development of DSPS and CPS? (3) Do the relationships between IR, DSPS and CPS change over time?

4. Methods

4.1. Sample

The sample of the study was drawn from 3rd to 11th grade students (aged 9–17) in Hungarian primary and secondary schools. There were 300–400 students in each cohort. The proportion of boys and girls was about the same. Some technical problems occurred during online testing resulting in data loss (completely at random). Participants who had more than 50% missing data on any of the problem solving measures were excluded ($N=316$). The final sample contained data from 2769 students for IR and DSPS measures, whereas 788 students' (random subsample of 5–11th graders only) data sets were available on CPS.

Verbal analogies	CHAIR : FURNITURE = DOG : ?				
	a CAT	b ANIMAL	c SPANIEL	d TABLE	e DOGHOUSE
Number analogies	20→32 :: 8→20 :: 11→_____				

Fig. 1. Examples of tasks in the inductive reasoning test (the original items were in Hungarian).

4.2. Design

The tests used for the study cover IR, DSPS and CPS. Different versions with different levels of item difficulty of IR and DSPS tests were used, which varied by school grade. The test versions for both constructs contained anchor items allowing achievement scores to be represented on a single scale in each case. All students took an IR and a DSPS test, 5–11th grade students additionally completed the CPS test.

The IR test was constructed at three levels of difficulty: The first level test was administered to Grade 3 students, the second level test to Grade 4 students and the third level test to students in Grades 5–11. Three different levels of DSPS test were used in the study. The first level DSPS test was administered to Grade 3–5 students, the second level to Grade 6–8 students and the third level to Grade 9–11 students. Finally, one age-independent version of the CPS test was used. That is, everybody, regardless of grade had to complete the same CPS test.

4.3. Materials

The IR tests comprised both open-ended and multiple-choice items (see Fig. 1). The first level IR test contained anchor items to the second level IR test (altogether 15 items), which contained anchor items to the third level IR test as well (8 items). In addition to these anchor items, ‘fat anchors’ were also used to connect all three levels of the test (26 items). The test for the oldest cohort (Grade 5–11) comprises three subtests: number analogies (14 items), number series (16 items) and verbal analogies (28 items; see Csapó, 1997). The three versions of the IR test consisted of a total of 72 items (Levels 1, 2 and 3: 48, 49 and 50 items, respectively).

DSPS was measured by two different types of tasks. Approximately 80% of the items were in multiple-choice format while the remaining items were student-constructed-response questions. With respect to the amount of information given, the DSPS test comprised three types of problems: (1) problems where all the information needed to solve the problem was given at the outset; (2) incomplete problems requiring the use of additional information previously learnt at school as part of the National Core Curriculum, and finally (3) incomplete problems requiring the use of additional information that was not learnt at school and needed to be retrieved from real-life knowledge. At each level of difficulty, the pages of the DSPS text booklets were divided into two columns. The left column presented information in realistic formats (e.g., map, picture, and drawing), whereas the right column introduced the story of a family trip or a class excursion and prompted students to solve problems (e.g., using the information provided and supplementing it with school knowledge) as they would arise during the trip (see Fig. 2). The DSPS tests consisted of 37 questions (Level 1: 17; Level 2: 22 and Level 3: 20) in total. The tests further contained anchor items linking the different levels: Between the first and second levels (9 items); between all three levels (3 items) and between the second and third levels (13 items) of the DSPS tests. Some of the items were included only at the first or the third level of the test (8 and 4 items, respectively).

The delivery guy also brought us a voucher entitling us for a 20% discount. When our pizzas arrived, they had such a marvellous smell that Mom and Dad also decided to have pizza for lunch. Dad sat in his car, took the voucher and bought a large “Forest’s captain” pizza for himself and a smaller pineapple pizza for Mom. What’s the minimum amount of money Dad had to take to the pizzeria?

CPS was measured by a set of seven tasks created in accordance with the MicroDYN approach (see Greiff et al., 2012; Wüstenberg et al., 2012). At the beginning, participants were provided with instructions including two practice tasks. Subsequently, participants had to explore an unfamiliar system, find out how variables were interconnected, and represent their knowledge in a situational model (knowledge acquisition; Funke, 2001). In addition, they had to control the system by reaching given target values (knowledge application; see Greiff et al., 2012; see Fig. 3).

4.4. Procedure and scoring

Test completion was divided into three sessions, each lasting approximately 45 min. In Session 1, students (in Grades 5–11) worked on the CPS test. In Session 2 students had to complete the DSPS test, and finally in Session 3 an IR test and a background questionnaire on demographical data were administered. The lengths of the breaks separating the sessions varied between classes.

The tests were administered in specially equipped computer rooms using the TAO platform (*Testing Assisté par Ordinateur*; Computer-Based Testing; see Farcot & Latour, 2008). The IR, DSPS and CPS tests were graded dichotomously. Full credit was given for a completely correct answer, whereas no credit was given if the answer contained a mistake.

Apród pizzák

megnevezés	20 cm	26 cm	32 cm	40 cm
Sajtos pizzaszósz, sajt, paradicsomlekvár	890	850	990	2890
Sonkás pizzaszósz, sonka, sajt	890	850	990	2890
Gombás pizzaszósz, gomba, sajt	890	850	990	2890
Kolbásos pizzaszósz, kolbász, sajt	890	850	990	2890
Szalámi pizzaszósz, szalámi, sajt	890	850	990	2890
Canbi pizzaszósz, sonka, trópusi gyümölcsök, sajt	890	850	990	2890
Erdő kapitánya pizzaszósz, sonka, gomba, sajt	890	850	990	2890
So-ku pizzaszósz, sonka, kukorica, sajt	890	850	990	2890

Főnemes pizzák

megnevezés	20 cm	26 cm	32 cm	40 cm
Hawaii pizzaszósz, sonka, ananász, kukorica, sajt	750	950	1090	3390
Sant-goku pizzaszósz, sonka, gomba, kukorica, sajt	750	950	1090	3390
Spencer tejföl, fűttojás, bacon, mozzarella	750	950	1090	3390
Mexikói chiliszósz, sonka, bab, kukorica, pfefferoni paprika, sajt	750	950	1090	3390

KOSÁR TARTALMA

Összesen: 0 Ft

*+ csomagolási és száll. díj

Név: _____
 Város: _____
 Cím: _____
 Telefon: _____
 E-mail: _____

Megjegyzés: _____

jegyezd meg a rendelésem

AKCIÓ!!!
EGYET FIZET, KETTŐT KAP.***

* Kiszállítási díj: 180 Ft. A doboz ára 70 Ft.
 ** Az ajánlat kizárólag a 28cm-es pizzára vonatkozik.

Másnap délelőtt átjött négy haverom, 11-kor már nagyon éhesek voltunk, rendeltünk egy-egy pizzát. Anna és Juli közösen kértek egy sonkás pizzát, a fiúk pedig egy-egy kicsi gombásat, én egy közepes mexikóit. Ez vajon mennyibe kerültek?

A: 3360Ft B: 3570Ft
 C: 3780Ft D: 2460Ft

A pizzafutár hozott egy 20%-os engedményre jogosító ajándékkupont is. Miután megérkezett a pizzánk, olyan jó illata volt, hogy anyuék is úgy döntöttek, ők is pizzát esznek ebédre. Apa beült a kocsiba, vitte a kupont és hozott magának egy nagyobb Erdő kapitánya pizzát, anyának pedig egy kicsit kisebb ananászos pizzát. Mennyi pénzt kellett apunak legalább vinni a pizzériába?

A: 1400Ft B: 1700Ft
 C: 2100Ft D: 2400Ft

Egész jól jártak a végén!

Fig. 2. Examples for tasks in the DSPS test (On the left you can place your order and you can see different pizzas with different prices, size and promotions. The story is on the right: The next day four of my friends came over. We were already very hungry at 11 a.m. and ordered some pizzas. Anna and Julia ordered a pizza with ham topping together, whereas the boys ordered a small pizza with mushroom each and I asked for a medium Mexican pizza. How much did this cost?/How much was the bill in total?)

Reach the given target area in no more than four steps! Round 5 63

blue chips -- - 0 + ++

green chips -- - 0 + ++

red chips -- - 0 + ++

Help Reset Apply!

Royal 16 [13-15]

Grande 18 [15-17]

Nobilis 14 [17-19]

Finish task

Modell

```

    graph LR
      blue[blue chips] --> Royal[Royal]
      blue --> Grande[Grande]
      green[green chips] --> Grande
      green --> Nobilis[Nobilis]
      red[red chips] --> Nobilis
    
```

Fig. 3. Screenshot of the MicroDYN task Gaming Night. [The original items were in Hungarian. The controllers of the input variables range from “–” (value = –2) to “+” (value = +2). The current values and the target values of the output variables are displayed numerically (e.g., current value for Royal: 16, target value: 13–15) and graphically (current value: dots; target value: red line). The correct model is shown at the bottom of the figure. See Greiff et al. (2013).

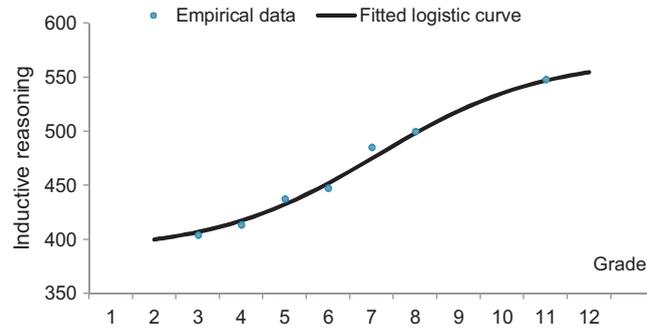


Fig. 4. Developmental curve of inductive reasoning.

Rasch's model was used for scaling the data (Bond & Fox, 2001), and then linear transformation of the logit metric was chosen. The mean of 8th graders was set to 500 with a standard deviation of 100. A four-parameter logistic equation [$F(x) = ((A - D)/(1 + ((x/C)^B)) + D$; A, minimum asymptote; B, hill slope; C, inflection point; D, maximum asymptote] was used for the curve fitting procedures (Molnár & Csapó, 2003) to obtain information on developmental trends. This is an established method of describing developmental processes (Yeagers, Shonkwiller, & Herold, 1996). The coefficient of determination (R^2) was computed to determine how well the model described the data.

5. Results and discussion

5.1. Descriptive statistics

While internal consistencies of the IR, DSPS and CPS tests were high ($IR_{\text{level } 1-3} \alpha = .93, .94, .95$; $DSPS_{\text{level } 1-3} \alpha = .73, .82, .65$, $CPS \alpha = .92$, respectively), there was a noticeable drop in reliability in the Level 3 DSPS test. The grade level analyses reveal that the results for Grades 9 and 10 showed an increased probability of measurement errors (e.g., $DSPS_{\text{grade } 9,10} \alpha = .57, .57$), while the reliability of the same test in Grade 11 (e.g., $DSPS_{\text{grade } 11} \alpha = .72$) proved to be higher. For this reason, we excluded the data for Grades 9 and 10 from all further analyses.

5.2. Development of inductive reasoning, domain-specific problem solving and complex problem solving

As revealed by the curve fitting procedure the logistic curve fitted the empirical data very well for inductive reasoning; the coefficient of determination was good ($R^2 = .99$). The point of inflexion (EC50 – half maximal effective concentration) was at the age of 13.1, indicating that a significant turn occurred in Grade 7, namely, development slowed down. Across all grades, development of inductive reasoning was significant; however, the pace of development was relatively slow, at about one quarter of a standard deviation per year. The fastest development (38 points) occurred between Grades 6 and 7 on the 500 (100) scale (see Fig. 4).

For DSPS, the fitted logistic curve also described the empirical data well ($R^2 = .95$). The point of inflexion (EC50) was at the age of 13.3. That is, the development slowed down after Grade 7. In elementary school, the development of DSPS was noticeable across all grades; however, this continuous development stopped in secondary school at the age of 15. The rate of development was relatively slow, at about one sixth standard deviation per year and varied between grades (see Fig. 5). No development was detectable between Grades 4 and 5. The fastest development occurred from Grade 7 to 8; it was more than

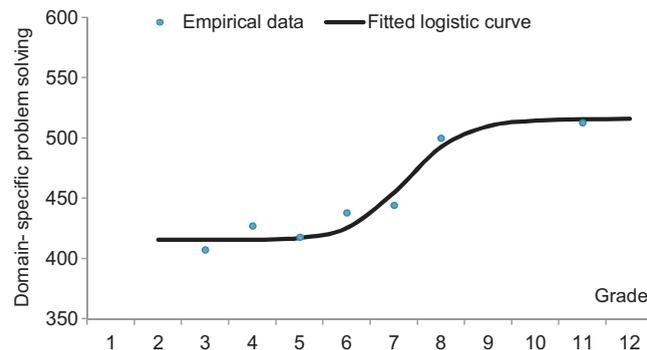


Fig. 5. Developmental curve of domain-specific problem solving.

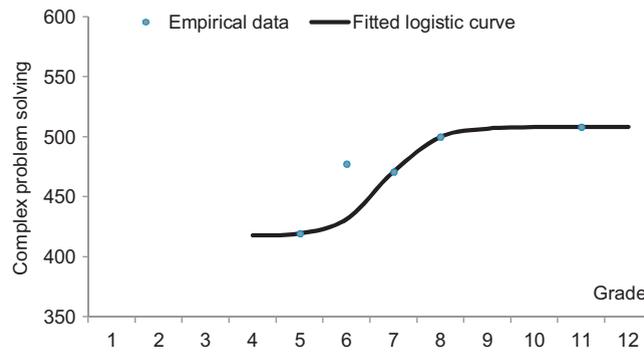


Fig. 6. Developmental curve of complex problem solving.

four times greater (56 points) than the average value (13 points/year) computed for the period from Grade 3 to Grade 11. To sum up, according to the empirical data, DSPS developed relatively intensively at a specific point in time during compulsory education but it developed at a slower rate than did IR.

The fitted logistic curve represented the empirical data adequately ($R^2 = .91$; see Fig. 6) in the domain of CPS. With the results of Grade 6 students excluded from the analyses, the fit was perfect ($R^2 = 1.00$). The point of inflexion (EC50) was at the age of 12.8, indicating that a significant change in development occurred in Grade 7. After Grade 7 the pace of development slowed down in CPS skills on average. In elementary school (up to Grade 8) there was noticeable development in CPS almost throughout the period. However, the developmental trend changed after Grade 8. From Grade 5 to 8 the rate of development was at about one quarter standard deviation per year and varied between different grades (see Fig. 6). No development was detectable between Grades 6 and 7; however, the behaviour of Grade 6 students calls for further research because the currently available empirical data do not account for this phenomenon. The fastest development was observed between Grades 5 and 7.

The fitted logistic curves showed similar trends in the development of DSPS and CPS during compulsory schooling. The slowest development took place during lower elementary school years and the fastest in secondary school. That is, in both cases the greatest development was observed during the upper elementary school years from Grade 5 to 8.

To summarize the developmental trends in IR, DSPS and CPS as thinking skills, the results were in line with the findings of previous studies (e.g. Adey et al., 2007), namely, thinking skills develop over time, and their development spans several years offering opportunities for the enhancement and fostering of these skills. As the stimulation of thinking skills is most efficient when their development is still in progress, especially when they are in a fast-growing phase, it is important to identify this sensitive period in students' lives. According to our results, the development of IR, DSPS and CPS follow a regular developmental trend; each can be described with a logistic curve. For all three constructs, the period of fastest growth was observed in Grade 7. Thus, this is the most effective time to enhance students' IR, DSPS and CPS skills. Further, the extrapolation of the fitted logistic curves indicates that substantial development took place before the 3rd Grade and some improvement of IR, DSPS and CPS can also be expected after Grade 11.

The pace of development is relatively slow in every area but there are noticeable differences between the curves. In the area of IR the rate of development is about a quarter of a standard deviation per year, in the areas of DSPS and CPS the corresponding rate is about one sixth of a standard deviation per year and varies between grades, as mentioned above. These trends confirm the results on the relatively slow development of thinking skills found in the literature (see e.g. Csapó, 1997), suggesting that there is a lack of direct and explicit stimulation of IR, DSPS and CPS in schools (de Konig, 2000; Molnár, 2011). More specifically, development is not encouraged by explicit instruction but simply occurs spontaneously as a 'by-product' of schooling.

5.3. The relationships between inductive reasoning, domain-specific problem solving and complex problem solving

The bivariate correlations between IR, DSPS and CPS were moderate ranging from .35 to .44 (Fig. 7). The relationships proved to be similar between IR and either DSPS or CPS ($r = .43$ and $.44$, $p < .01$, respectively) and they were significantly stronger ($z = 1.80$, $p < .05$) than the correlation between DSPS and CPS ($r = .35$, $p < .01$). Partial correlations were significantly lower as all bivariate relationships were influenced by the third construct ($r_{IR,DSPS} = .26$; $r_{IR,CPS} = .33$; $r_{DSPS,CPS} = .26$, $p < .01$ respectively). These partial correlation coefficients were of the same strength ($p > .05$).

Our results show that IR, DSPS, and CPS are not identical but correlated constructs. The degrees of correlations were on the whole moderate confirming theoretical studies (see e.g. Wirth & Klieme, 2004) pointing out that all of these skills involve cognitive abilities indispensable for generating and applying different rules. IR as a basic thinking skill and as a skill applied in information processing during problem solving had a stronger effect on DSPS and CPS than DSPS and CPS had on each other confirming previous research results (e.g. Klauer, 1996; Hamers et al., 2000), namely that IR influences the success of knowledge acquisition and application as it plays a crucial role in domain-specific and complex problem solving.

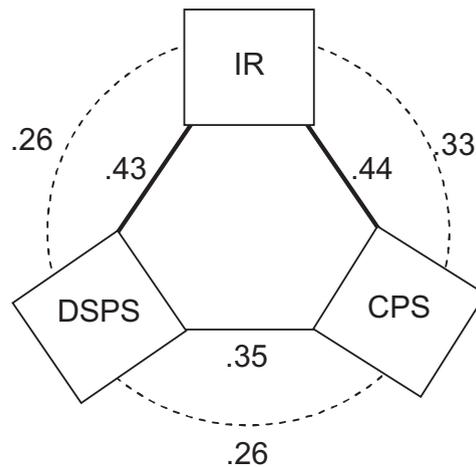


Fig. 7. Relations between IR, DSPS and CPS (all coefficients are significant at $p < .01$).

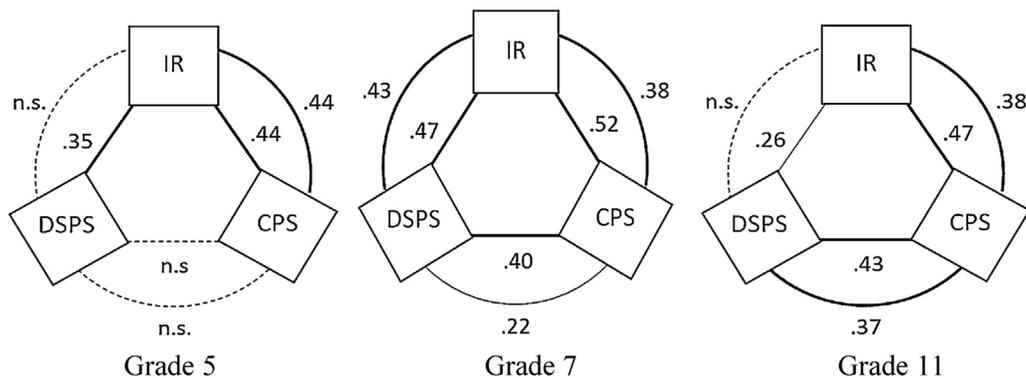


Fig. 8. Changes in relationships between IR, DSPS and CPS in Grades 5, 7 and 11 (all coefficients are significant at $p < .01$).

5.4. Changes in relationships between inductive reasoning, domain-specific problem solving and complex problem solving across school grades

Based on our analyses of reliability and goodness of fit, three different cohorts were selected to compare relationships between IR, DSPS, and CPS: 5th, 7th and 11th graders. As shown by our data, 5th graders' skills show some development but they have not entered the fast-growing phase, 7th graders are in the fast-growing developmental phase, and, finally, 11th graders have left behind the fast-growing phase and approach the end of compulsory education.

The correlation patterns differed between the three cohorts. The strengths of the correlation coefficients were more homogeneous within grades than across grades. On the whole, the strength of the relationship between IR and CPS proved to be the highest and the most stable over time. The correlation patterns are more similar in the lowest and highest of the three grades than in Grade 7 in the middle, where both the bivariate and partial correlations were the highest.

The relationship between DSPS and CPS became stronger over time; there were no significant correlations in lower grades but a moderate but significant correlation tended to be observed in higher grades. With the analyses on all nine relationships (between three constructs in three groups) included, partial correlations were significantly lower than bivariate correlations in only three cases, indicating that controlling for the third construct did not generally lower the strength of the relationship of the other two. For example the relationship between IR and CPS is not due to students' level of DSPS skills.

The relative stability of correlation coefficients in connection with IR and CPS can be explained by analysing the role of IR in other cognitive processes. The basic mechanisms of IR such as comparing objects and their attributes, finding similarities and dissimilarities between them, or generating rules based on observation can be identified in CPS as well. As the correlations in Fig. 8 indicate, the relationships between IR and CPS are especially high and remain largely unchanged after controlling for DSPS, in contrast to the relationships between IR and DSPS. However, this observation is holds for Grade 5 and 11 only, and cannot be seen in Grade 7.

The role of IR in DSPS appears to be independent of the growing amount of information and knowledge students learn in and outside the classroom; it depends only on the pace of development of DSPS and IR. When IR is in the fastest growing phase (Grade 7), it has the highest influence on DSPS skill level. Approaching the data from another direction, the strengths

of the correlations between IR and DSPS are the same before and after the fast-growing phase regardless of the amount of knowledge acquired at school.

The increasing correlation between DSPS and CPS can be explained by the fact that the strategies used in DSPS and CPS situations become more similar over time. DSPS is based on knowledge application, whereas CPS is a prerequisite to gaining and applying new knowledge which is largely represented in DSPS. That is, the mechanisms operated by DSPS and CPS are getting closer over time. If CPS is fostered, DSPS will be fostered as a 'by-product' of the developmental program and the other way round. Even more generally, if we foster one of the three constructs, the other two will be fostered as well as a secondary effect of the developmental program and we can achieve the greatest effect with such a developmental program in Grade 7.

6. Conclusions

Educational large-scale assessments focus explicitly on students' achievement in several broad content domains, but the implicit goal is to find ways of making education more effective. This is particularly relevant in today's society, in which the content of applicable knowledge changes rapidly (de Konig, 2000; Adey et al., 2007) and students frequently encounter new and unknown challenges. Nowadays students need to possess and use different skills and knowledge than they used to rely on in the slowly changing, static societies previously. As a result, the study of cross curricular thinking skills came to the forefront in which knowledge creation, organization and transfer play an important role as well. From educational point of view the assessment of these skills are important because their role can be detected in several learning tasks and developmental processes.

Nevertheless, the stimulation of thinking skills such as inductive reasoning or problem solving is not pursued explicitly in schools (de Koning, Hamers, Sijtsma, & Vermeer, 2002) as teaching and learning processes are often assumed to implicitly foster higher order thinking skills. However, research has shown that there are additional ways to significantly and effectively develop thinking skills, for instance by explicit training (Molnár, 2011) or by enriching school materials and modifying teaching methods (e.g. Adey & Shayer, 1994; Shayer & Adey, 2002).

The results of our study support the hypothesis that development of IR, DSPS and CPS takes place mostly during compulsory schooling and spans several years offering ample opportunity to explicitly foster these higher order thinking skills. The role of IR in the development of DSPS proved to be significant, indicating that IR has its special contribution to the knowledge acquisition phase of DPS. The strengths of the relationships between IR, DSPS and CPS changed over time signalling a qualitative change in the development of cognition.

When solving domain-specific problems, problem solvers need to combine knowledge acquired in and outside school in order to reach the desired solutions relying on previously acquired knowledge within a specific domain. A different approach is necessary in complex problem solving, in which the problem solver needs to use domain general cognitive processes instead of content knowledge and rote learning in order to cope with non-transparent and new situations (Buchner, 1995). These differences surface in our results through the stronger partial correlation between CPS and IR than between DSPS and IR. However, the data from the present survey are not consistent on this issue, as the youngest and oldest sample showed a similar pattern of relationships, whereas the middle sample deviated from this pattern. This may be the results of a structural reorganization of cognitive processes at this particular age. Further research is needed to identify the causes of this irregularity.

The general implication of our results is that the most effective period of intervention is the same for all three constructs, it is between Grades 6 and 8. This is, then, the sensitive period, i.e., the most effective time to enhance students' IR, DSPS and CPS skills. Specifically, interventions affecting one skill may affect the other two as well. Therefore, we may assume that if IR is fostered, DSPS and CPS will be developed indirectly as well. There are two consequences of this result. First, it supports the conclusions emphasized by prior research, namely the importance of developing inductive reasoning as it is a major cognitive tool in knowledge acquisition and application (Hamers et al., 2000) as well as in problem solving (Klauer, 1996). Second, it draws attention to the issue of developing and introducing special methods for CPS enhancement, such as guided discovery, or using CPS tasks as assessment and training tools for domain-general knowledge acquisition and application skills that foster students IR and domain-specific knowledge acquisition and application skills. Thus, IR, DSPS and CPS play a central role in gaining a deeper understanding of what happens in the classroom, which, in turn, suggests that these thinking skills should become an integral part of school agendas (de Konig, 2000; Resnick, 1987) and should be incorporated into a broad range of school-related learning activities.

As we have seen, the problem solving strategies used in DSPS become more and more similar to the strategies used in CPS problems, thus after a while the same mechanisms, the same knowledge acquisition and application skills are made use of when solving different kinds of problems. This means that the role of knowledge and experience in specific content areas in solving problems decreases over time; these components are helpful but become less necessary for constructing abstract schemas (Novick & Bassok, 2005).

The present study contributes to the recently re-opened debate on the role of general-purpose and cross-curricular abilities in achieving successful participation in the 21st century's western society and on the limitations associated with understanding only specific abilities such as reading, writing, and math. We conclude by emphasizing the potential lying in reasoning skills such as inductive reasoning, domain-specific problem solving, and complex problem solving as educationally relevant constructs that can and should be fostered successfully over several years during compulsory schooling. However,

efforts in this direction should not be limited to domain-specific abilities, but additionally need to focus on other reasoning skills because fostering the development of thinking skills is one of the most important and most challenging educational goals currently ahead of us.

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